



Relativistic Quantum Transport in Graphene Systems

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07/09/2015
Final Report

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REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
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1. REPORT DATE (DD-MM-YYYY) 14-07-2015		2. REPORT TYPE Final Performance		3. DATES COVERED (From - To) 15-04-2012 to 14-04-2015	
4. TITLE AND SUBTITLE Relativistic Quantum Transport in Graphene Systems				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER FA9550-12-1-0095	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Ying Cheng Lai				5d. PROJECT NUMBER	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) ARIZONA STATE UNIVERSITY 660 S MILL AVE STE 312 TEMPE, AZ 85281 US				8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) AF Office of Scientific Research 875 N. Randolph St. Room 3112 Arlington, VA 22203				10. SPONSOR/MONITOR'S ACRONYM(S) AFOSR	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT A DISTRIBUTION UNLIMITED: PB Public Release					
13. SUPPLEMENTARY NOTES					
14. ABSTRACT <p>The principal Objective of the project was to exploit relativistic quantum manifestations of classical chaos in graphene and two-dimensional Dirac fermion systems. Methods were developed to solve the Dirac equation in arbitrary domains. New phenomena uncovered include relativistic quantum scarring, chiral scars, chaos-based quantum control, and chaos-regularized relativistic quantum tunneling, etc. The AFOSR support helped create a new field of interdisciplinary research: Relativistic Quantum Chaos, which studies the relativistic quantum manifestations of classical chaos with applications with implications to the development of next generation of nanoscale electronic devices and circuits based on graphene and alternative two dimensional Dirac materials. The AFOSR project resulted in 20 refereed-journal papers, including papers in high-impact journals such as Physical Review Letters, and provided PI with the opportunity to supervise a number of PhD students: two graduated, one to graduate in 2016, and two ongoing. PI gave about a dozen plenary lectures, seminars, and colloquiums all over the world on relativistic quantum chaos.</p>					
15. SUBJECT TERMS electromagnetic wave scattering, chaos theory, waveform design					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON Ying Cheng Lai
a. REPORT U	b. ABSTRACT U	c. THIS PAGE U			19b. TELEPHONE NUMBER (Include area code) 480-965-6668

Final Report

This Final Report summarizes activities under the Air Force Office of Scientific Research (AFOSR) Grant No. FA9550-12-1-0095 entitled “RELATIVISTIC QUANTUM TRANSPORT IN GRAPHENE SYSTEMS” from 15 April 2012 to 15 April 2015. PI is Ying-Cheng Lai from Arizona State University (ASU).

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1 Objectives

- Klein tunneling, abnormal electron paths, and extreme conductance fluctuations in graphene quantum point contacts.
- Control of quantum transport in nanostructures by classical chaos.
- Relativistic quantum Darwinism and applications.

*The AFOSR support helped create a new field of interdisciplinary research: **Relativistic Quantum Chaos**, which studies the relativistic quantum manifestations of classical chaos with applications in graphene and alternative two-dimensional Dirac material systems.*

2 List of Publications

1. X. Ni, L. Huang, Y.-C. Lai, and L. M. Pecora, "Effect of chaos on relativistic quantum tunneling," *Europhysics Letters* **98**, 50007, 1-6 (2012). This paper was highlighted in *Europhysics News*, Vol. 43, No. 6 (2012).
2. L. Ying, L. Huang, Y.-C. Lai, and C. Grebogi, "Conductance fluctuations in graphene systems: The relevance of classical dynamics," *Physical Review B* **85**, 245448, 1-7 (2012).
3. X. Ni, L. Huang, Y.-C. Lai, and C. Grebogi, "Scarring of Dirac fermions in chaotic billiards," *Physical Review E* **86**, 015702, 1-8 (2012).
4. X. Ni, Y.-C. Lai, and W.-X. Wang, "Emergence of scaling associated with complex branched wave structures in optical medium," *Chaos* **22**, 043116, 1-11 (2012).
5. H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, "Chiral scars in chaotic Dirac fermion systems," *Physical Review Letters* **110**, 064102, 1-5 (2013). This work was selected by PRL Editors as an Editors' Suggestion.
6. L. Huang, R. Yang, Y.-C. Lai, and D. K. Ferry, "Lead-position dependent regular oscillations and random fluctuations of conductance in graphene quantum dots," *Journal of Physics: Condensed Matters* **25**, 085502, 1-7 (2013).
7. L. Ying, L. Huang, Y.-C. Lai, and Y. Zhang, "Effect of geometrical rotation on conductance fluctuations in graphene quantum dots," *Journal of Physics: Condensed Matters* **25**, 105802, 1-7 (2013).
8. R. Yang, L. Huang, Y.-C. Lai, C. Grebogi, and L. M. Pecora, "Harnessing quantum transport by transient chaos," *Chaos* **23**, 013125, 1-9 (2013).
9. G.-L. Wang, L. Ying, Y.-C. Lai, and C. Grebogi, "Quantum chaotic scattering in graphene systems in the absence of invariant classical dynamics," *Physical Review E* **87**, 052908, 1-9 (2013).
10. X. Ni, L. Huang, L. Ying, and Y.-C. Lai, "Relativistic quantum tunneling of Dirac fermions in nonhyperbolic chaotic systems," *Physical Review B* **87**, 224304, 1-10 (2013).
11. G.-L. Wang, L. Huang, Y.-C. Lai, and C. Grebogi, "Nonlinear dynamics and quantum entanglement in optomechanical systems," *Physical Review Letters* **112**, 110406, 1-6 (2014).
12. L. Ying, Y.-C. Lai, and C. Grebogi, "Quantum manifestation of synchronization transition in optomechanical systems," *Physical Review A* **90**, 053810, 1-5 (2014).
13. L. Ying, G.-L. Wang, L. Huang, and Y.-C. Lai, "Quantum chaotic tunneling in graphene systems with electron-electron interactions," *Physical Review B* **90**, 224301, 1-14 (2014).

14. L. Huang, Y.-C. Lai, H.-G. Luo, and C. Grebogi, “Universal formalism of Fano resonance,” *AIP Advances* **5**, 017137, 1-18 (2015).
15. Y.-Y. Park, Y.-H. Do, S. Altmeyer, Y.-C. Lai, and G.-W. Lee, “Early effect in time-dependent, high-dimensional nonlinear dynamical systems with multiple resonances,” *Physical Review E* **91**, 022906, 1-7 (2015).
16. H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, “Superpersistent currents and whispering gallery modes in relativistic quantum chaotic systems,” *Nature Scientific Reports* **5**, 8963, 1-8 (2015).
17. S. Altmeyer, Y.-H. Do, and Y.-C. Lai, “Transition to turbulence in Taylor-Couette ferrofluidic flow,” *Nature Scientific Reports* **5**, 10781, 1-12 (2015).
18. S. Altmeyer, Y.-H. Do, and Y.-C. Lai, “Ring bursting behavior en route to turbulence in quasi two-dimensional Taylor-Couette flows,” submitted to *Physical Review E*.
19. P.-P. Li, Z.-G. Huang, L. Huang, and Y.-C. Lai, “Lyapunov exponent of Sinai billiard,” submitted to *EuroPhysics Letters*.
20. R. Bao, L. Huang, Y.-C. Lai, and C. Grebogi, “Conductance fluctuations in chaotic bilayer graphene quantum dots,” submitted to *Physical Review E*.

3 Accomplishments and New Findings

3.1 Solutions of Dirac equation, relativistic quantum tunneling and scarring

3.1.1 General method for solving the Dirac equation in closed domains and relativistic quantum scarring

Given a closed Hamiltonian system that exhibits fully developed chaos in the classical limit, one might expect the quantum wavefunctions associated with various eigenstates to be more or less uniform in the physical space. This had been thought to be quite natural because of ergodicity associated with chaos in the classical phase space. The notion of uniform wavefunctions was nevertheless proven to be wrong about three decades ago, where strongly non-uniform eigenfunctions were discovered by McDonald and Kaufman in their study of the eigen-solutions of the Schrödinger equation in the chaotic stadium billiard. A systematic study was subsequently carried out by Heller, who established the striking tendency for wavefunctions to concentrate about classical unstable periodic orbits, which he named *quantum scars*. Semiclassical theory was then developed by Bogomolny and Berry, providing a general understanding of the physical mechanism of quantum scars. It should be noted that, the phenomenon was deemed counterintuitive and surprising solely because of *chaos*, as the phase space of an integrable system is not ergodic so that the quantum wavefunctions are generally not expected to be uniform. Quantum scarring is one of the most remarkable phenomena in modern physics, and has been an active area of research.

Previous works on scarring focused on non-relativistic quantum systems described by the Schrödinger equation. A fundamental issue was whether, in a closed chaotic domain, relativistic quantum particles obeying the Dirac equation can scar. This issue was partially addressed by PI’s group in the context of closed graphene confinements, where signatures of quantum scars were identified in the patterns of the local density of states. The framework used in the study, however, was based on the tight-binding Hamiltonian and the non-equilibrium Green’s function derived still from the Schrödinger equation. The question of whether truly Dirac fermions, particles strictly obeying the Dirac equation, can scar in chaotic billiards had not been addressed.

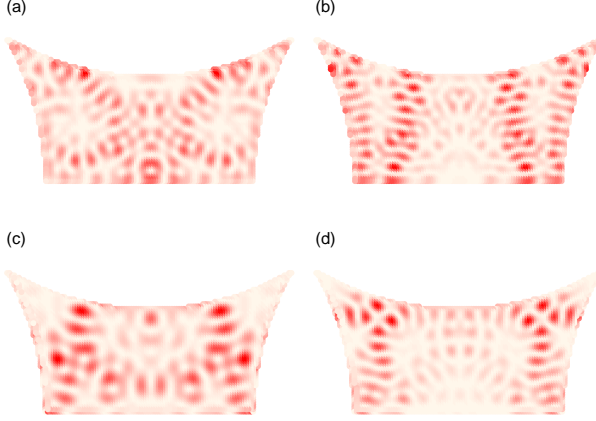


Figure 1: Examples of eigenstates of the Dirac equation for a bow-tie chaotic billiard. The maximum height (vertical distance from tip to base) is set to 1, and the distance between two tips is 2. Panels (a-d) are for $E = 36.1335, 43.0190, 27.9300,$ and 36.2729 , respectively. Panels (a) and (b) show the first component of the Dirac spinor, while (c) and (d) show the second component.

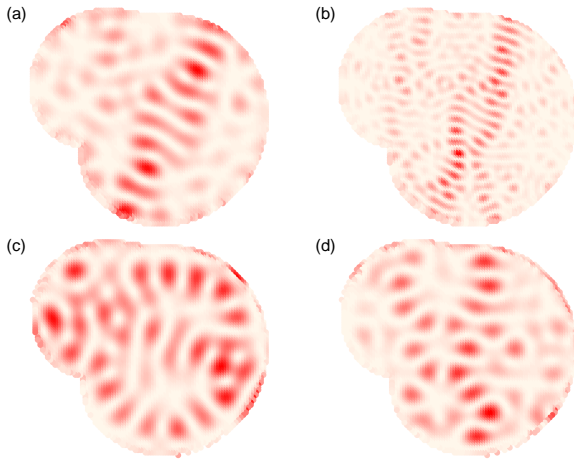


Figure 2: Examples of eigenstates of the Dirac equation for the chaotic “Africa” billiard. The domain is confined within a rectangular box $x \in [-0.99, 1.35]$ and $y \in [-1.22, 0.92]$. Panels (a-d) are for $E = 15.0468, 32.7638, 13.4838,$ and 15.2402 , respectively. Panels (a) and (b) show the first spinor component, while (c) and (d) show the second spinor component.

Prior to our work, a general method for solving completely the Dirac equation in *closed* system of *arbitrary* geometry had not existed. The main reason was that, although the Dirac equation is the cornerstone of relativistic quantum mechanics and quantum electrodynamics, solutions were focused on free space and perturbation types in situations where relativistic quantum behaviors occur. Prior to the discoveries of graphene and topological insulators, it was not generally thought that relativistic quantum mechanics would be relevant to solid state devices. As a result, there had been little interest in studying the Dirac equation in *finite domains*. The only exception was the work of Berry and Mondragon, who in 1987 developed a boundary-integral type of method to solve the energy levels (eigenvalues) of a chaotic neutrino billiard. To obtain the eigenfunctions, a closed form of the boundary of the domain is needed.

Supported by AFOSR, we developed a general and efficient method to solve the Dirac equation for massless fermions in two-dimensional closed domains. An obstacle to obtaining a complete solution of the Dirac equation, which includes both eigenvalues and eigenfunctions, was the proper handling of the boundary conditions. We articulated an efficient discretization scheme and a physically meaningful approach to treating the boundary conditions, converting the Dirac equation into a set of matrix equations. In our method, the physical symmetries of the system are well preserved. To validate our method, we considered three types of representative geometric confinements, which include domains that generate integrable or chaotic motions in the classical limit, and calculated the complete spectra of eigenvalues and the associated eigenvector sets. In particular, in the case of integrable geometries for which analytic predictions of the eigenvalues and eigenvectors are available, we obtained excellent agreement between the numerical and

analytic results. For more general geometries including classically chaotic systems, the properties of the calculated eigenvalue spectrum, such as the energy level-spacing statistics, agree well with the known results for different symmetry classes. In fact, our method is capable of finding eigenstates of Dirac fermions under arbitrarily electrical potential profiles. Our matrix formulation can be applied directly to one-dimensional systems. Through a straightforward extension of the Dirac spinor to four components and a proper revision in the discretization and boundary constraints, the method can be extended to solving the Dirac equation in three dimensions as well. Our main finding was that the relativistic, spinor type of wavefunctions associated with Dirac fermions can be highly non-uniform in chaotic billiards, and truly relativistic quantum scars *do* exist. Examples of such scars are shown in Figs. 1 and 2.

A number of areas in physics can benefit from an efficient method for solving the Dirac equation. The most relevant area is graphene physics. Graphene ribbons exhibit a linear energy-momentum relation near a Dirac point in the energy-band diagram, which is a characteristic of relativistic quantum motion of massless fermions. In the presence of short-range potentials, two Dirac points are coupled together. It is thus of basic interest to investigate the behavior of pure Dirac fermions to distinguish them from those due to the coupling of two relativistic particles.

Details of this work can be found in

- X. Ni, L. Huang, Y.-C. Lai, and C. Grebogi, “Scarring of Dirac fermions in chaotic billiards,” *Physical Review E* **86**, 015702, 1-8 (2012).

3.1.2 Chiral scars in chaotic Dirac fermion systems

During the project period, we discovered a new class of quantum scars: chiral scars in chaotic Dirac fermion systems. This was motivated by the following question: are there characteristics of relativistic quantum scars which differ fundamentally from those associated with non-relativistic quantum scars? To make possible our search for such characteristics, we developed an *analytic* approach to calculating the eigenstates of massless Dirac fermions in a broad class of chaotic billiards by using the method of conformal mapping. In particular, for any shape in the class, a proper conformal mapping can transform it to a shape for which the solutions of the Dirac equation can be written down analytically. An inverse transform of the solutions thus leads to eigenstates in the original domain. This method allows us to calculate an unprecedentedly large number of eigenvalues and eigenstates with high accuracy. Taking advantage of this powerful method, we succeeded in identifying one such characteristic associated with the phase of the wavefunction. To explain this, we note that, in non-relativistic quantum systems, when a particle traverses one cycle along a scarred orbit, the associated quantum phase change is zero or 2π . However, when the various eigenstate solutions of the massless Dirac equation were examined, we found one subclass of scarred orbits for which one complete itinerary brings about a phase change of only π . In fact, it takes two cycles for the phase of the wavefunction to become 2π so that the wavefunction to return completely to its original value. This relativistic quantum phenomenon is originated from the chirality of the massless Dirac fermions, and consequently we named such scars *chiral scars*. It should be noted that, despite the emergence of chiral scars, majority of the scars are conventional in the sense that the phase change associated with one cycle is 2π . We developed a semiclassical theory to explain the physical origin of chiral scars, which are uniquely relativistic quantum scars and find no counterparts in non-relativistic quantum systems.

Details of this work can be found in

- H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, “Chiral scars in chaotic Dirac fermion systems,” *Physical Review Letters* **110**, 064102, 1-5 (2013). This work was selected by PRL Editors as an Editors’ Suggestion.

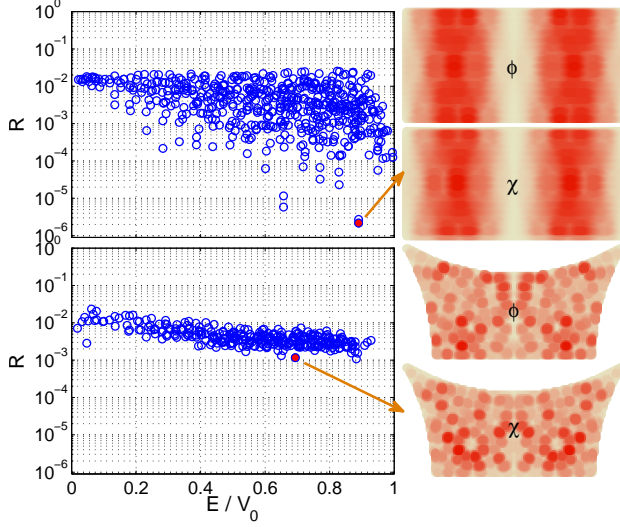


Figure 3: **Tunneling rates and LDS patterns for massless Dirac fermion in integrable and chaotic double-well systems**, where ϕ and χ are two components of the Dirac spinor. A unit system $\hbar = c = 1$ has been used in our calculations, in which the barrier height V_0 is about 60. For the rectangle double-well, the width and the height are set to 2 and 1, respectively. The bow-tie chaotic shape is obtained by cutting the rectangle with three arcs such that the cut parts are all 0.3 measured on both sides of the baseline and central vertical line of the rectangle. The theoretical ratio of the left well width to the barrier width is 24:1.

3.1.3 Relativistic quantum tunneling in Dirac fermion and graphene systems

In 2011, the remarkable phenomenon of chaos-regularized tunneling in non-relativistic quantum systems was uncovered, where classical chaos can suppress, significantly, the spread in the tunneling rate commonly seen in systems whose classical dynamics are regular. For example, for a system consisting of two symmetrical cavities connected by a one-dimensional potential barrier along the line of symmetry, when the classical dynamics in each cavity is integrable, for sufficiently large energy the tunneling rate can assume many values in a wide interval. Choosing the geometry of the cavity such that the classical dynamics become chaotic can greatly enhance and regularize quantum tunneling. Heuristically, this can be understood, as follows. When the potential barrier is infinite, each cavity is a closed system with an infinite set of eigenenergies and eigenstates. Many eigenstates are concentrated on classical periodic orbits, forming quantum scars. For classically integrable cavity, some stable or marginally stable periodic orbits can persist when the potential barrier becomes finite so that each cavity system is effectively an open quantum system. Many surviving eigenstates correspond to classical periodic orbits whose trajectories do not encounter the potential barrier, generating extremely low tunneling rate even when the energy is comparable with or larger than the height of the potential barrier. The eigenstates corresponding to classical orbits that interact with the potential barrier, however, can lead to relatively strong tunneling. In a small energy interval the quantum tunneling rate can thus spread over a wide range. However, when the classical dynamics is chaotic, isolated orbits that do not interact with the potential barrier are far less likely and, consequently, the states associated with low tunneling rates disappear, effectively suppressing the spread in the tunneling rate.

We addressed the question of whether chaos can regularize tunneling in relativistic quantum systems. To be concrete, we studied the motion of massless Dirac fermions in the setting of resonant tunneling to facilitate comparison with the non-relativistic quantum case. Our extensive computations revealed unequivocally the existence of surviving eigenstates that lead to extremely low tunneling rates. As for the non-relativistic quantum case, making the cavities classically chaotic can greatly regularize the quantum tunneling dynamics. Figure 3 shows the generalized tunneling rate R versus the normalized energy E/V_0 for a massless Dirac fermion in the double-well barrier system for two types of geometry: one classically integrable and another chaotic. For the integrable geometry, we observe the existence of states with extremely low tunneling rates, as indicated by the arrow in Fig. 3(a). These correspond to states localized nearly entirely in the left or right side of the potential barrier, which “survive” the tunneling process between the two sides, as indicated by the accompanying pattern of local density of states (LDS). In non-relativistic quantum transport, these are

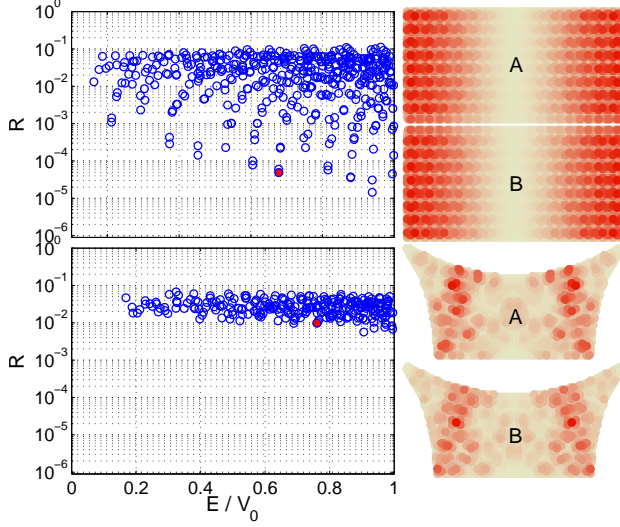


Figure 4: **Tunneling rates and LDS patterns in an integrable and a chaotic graphene double-well systems**, where A and B denote the two distinct types of atoms in a graphene unit cell. Tight-binding Hamiltonian is used. The barrier height is fixed at $V_0 \sim 0.67t$ (with t denoting the nearest-neighbor hopping integral). After converting to the $\hbar = c = 1$ unit system, the geometric measures and the barrier potential are the same as for the single Dirac fermion system.

effectively quantum pointer states. In relativistic quantum systems, we observe that both components of the underlying Dirac spinor exhibit a heavy probability concentration on orbits along which particles travel vertically back and forth on either side, parallel to the barrier. For the chaotic geometry, while signatures of pointer states can still be found, they are much weaker than those in the integrable counterpart, as shown in Fig. 3(b) and the accompanying LDS pattern.

For practical implications, we considered resonant tunneling devices made entirely of graphene and calculated the tunneling rate for different energy values. We obtained qualitatively similar results to those for massless Dirac fermions, as shown in Fig. 4. One unique feature for both the Dirac and graphene systems, which finds no counterpart in non-relativistic quantum tunneling devices, is the high tunneling rate in the regime where the particle energy is smaller than the height of the potential barrier. This is a manifestation of Klein tunneling. To explain the numerical findings, we developed a theory based on the concept of self energies and the complex energy spectrum of the non-Hermitian Hamiltonian for the “open” cavity.

Details of this work can be found in

- X. Ni, L. Huang, Y.-C. Lai, and L. M. Pecora, “Effect of chaos on relativistic quantum tunneling,” *Europhysics Letters* **98**, 50007, 1-6 (2012). This paper was highlighted in *Europhysics News*, Vol. 43, No. 6 (2012).

3.1.4 Relativistic quantum tunneling in nonhyperbolic chaotic systems

A challenging and fundamental problem in the field of quantum chaos is to understand quantum manifestations of chaotic behaviors in situations where the classical dynamics are *nonhyperbolic*. In classical Hamiltonian systems, nonhyperbolicity is characterized by coexistence of chaos and Kolmogorov-Arnold-Moser (KAM) tori in the phase space, hence the term “mixed phase space.” The problem had been addressed previously but mostly in the realm of nonrelativistic quantum mechanics governed by the Schrödinger equation. For example, in open Hamiltonian systems where the classical dynamics are chaotic scattering, presence of KAM tori can result in enhanced fluctuations of the semiclassical S-matrix elements or even lead to fractal fluctuations.

Development of graphene physics in the past decade stimulated a great deal of interest in relativistic quantum mechanics as applied to solid-state devices. Graphene systems, such as quantum dots that exhibit chaos in the classical limit, were studied with respect to issues such as the energy-level statistics, quantum

scars, and scattering dynamics. From the classical point of view, nonhyperbolic systems are generic while integrable and fully chaotic systems are the exception, as the latter correspond to the two opposite extreme cases in the spectrum of nonlinear Hamiltonian systems. Thus, in applications of graphene devices, nonhyperbolic dynamics are expected to be more common than integrable or chaotic dynamics, and this demands a good understanding of the relativistic quantum manifestations of nonhyperbolicity.

The linear energy-momentum relation in graphene systems, which is characteristic of relativistic quantum motion, however, holds only approximately near a Dirac point in the energy-band diagram. In addition, while the motion of electrons in graphene near a single Dirac point can be described by the Dirac equation, in realistic systems the coupling between motions associated with the coexisting Dirac points cannot be neglected. Thus, strictly speaking, study of chaotic graphene systems can lead only to a partial understanding of various phenomena in the emerging field of *relativistic quantum chaos*. To obtain a complete picture, fermion systems governed by the Dirac equation must be studied.

A paradigm of nonhyperbolic chaotic systems is the “mushroom” billiard for which a mathematical proof of nonhyperbolicity was obtained by Bunimovich in 2001. By placing a finite potential barrier along the vertical, symmetric line of the billiard, we effectively generate the setting of resonant tunneling where a particle can tunnel through the barrier from one side of the billiard to the other. We assumed that the particle is a massless Dirac fermion, whose motion is governed by the Dirac equation, and we focused on the effect of nonhyperbolicity on the tunneling rate $\gamma(E)$, the fundamental quantity characterizing the quantum tunneling dynamics, which depends on the particle energy E . Our main finding was that nonhyperbolicity leads to a “clustering” phenomenon where the majority of the values of $\gamma(E)$ fall into a narrow “band” in the (E, γ) plane, with relatively few values outside the band, as shown in Fig. 5. To understand this clustering phenomenon, we develop a theoretical framework based on the concept of self-energies to calculate the tunneling rate. We then analytically solved the Dirac equation both in one dimension and in two dimensions for a circular-ring type of tunneling system with integrable dynamics in the classical limit. Due to the presence of relatively few and *distinct* classical periodic orbits in the integrable component, the corresponding relativistic quantum states can have drastically different behaviors, leading to a wide spread of the values of the tunneling rate in the (E, γ) plane. However, the chaotic component provides significantly more quantum states for tunneling due to the infinite set of unstable periodic orbits embedded therein. These states are dynamically “similar” because of the ergodic nature of chaos, and this leads to the clustering of the values of the tunneling rate in a narrow band.

Details of this work can be found in

- X. Ni, L. Huang, L. Ying, and Y.-C. Lai, “Relativistic quantum tunneling of Dirac fermions in nonhyperbolic chaotic systems,” *Physical Review B* **87**, 224304, 1-10 (2013).

3.1.5 Effect of many-body interactions on relativistic quantum chaos

In quantum chaos, the most often studied setting was that of single-particle quantum dynamics. While there were previous studies of the interplay between many-body interactions and classical chaos, these were exclusively for non-relativistic quantum systems described by the Schrödinger equation. To investigate the effect of chaos on *relativistic* quantum systems with many-body interactions is an outstanding problem, yet it is not only fundamental to physics, but also important for the practical development of relativistic quantum devices based on graphene or alternative two-dimensional Dirac material systems.

In the project period, we studied quantum chaos in the presence of many-body interactions by using the standard Hubbard model. This paradigmatic model to treat interacting particles in a lattice was originally proposed in 1963 to describe the transition between conducting and insulating systems. For electrons in a solid, comparing with the conventional tight-binding model representing a single electron Hamiltonian, the

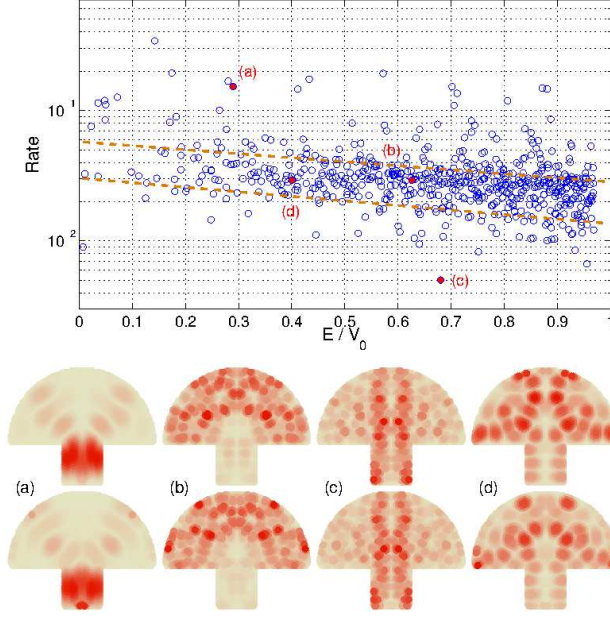


Figure 5: Tunneling rates versus the energy and representative eigenstates for mushroom billiard made of graphene. The dimensions of the graphene system and the barrier height and width are proportional to those of the nanoscale Dirac fermion tunneling system that we studied, but the first and the second rows of patterns represent, instead of Dirac spinors, the A and B atoms in the graphene unit cell. The eigenenergies and eigenstates are calculated by using the standard tight-binding Hamiltonian for the corresponding closed graphene system.

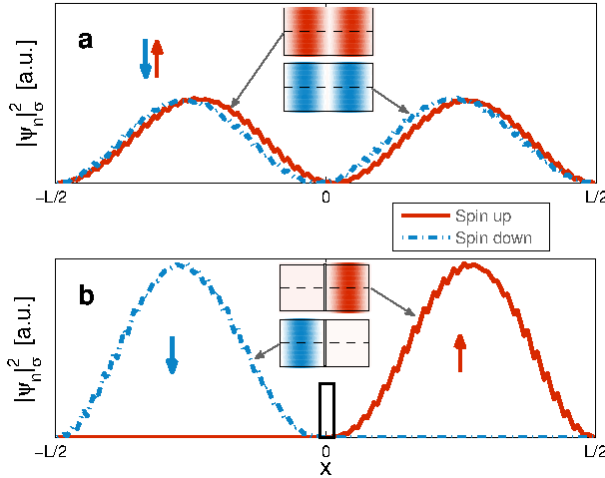


Figure 6: Effect of many-body interactions on eigenstates. Representative confined eigenstates associated with spin-up (red/solid curves) and spin-down (blue/dashed curves) electrons: (a) probability density profile of one eigenstate *without* a potential barrier, where it extends in both potential wells, (b) probability density profile of the eigenstate *with* a potential barrier at $x = 0$ (the gray rectangle). In (b), there is spin polarization, i.e., spin-up electrons reside in the right well and spin-down electrons reside in the left well. The insets in both panels show the corresponding patterns of local density of state in the entire domain.

Hubbard model contains a potential term to include the many-body effect through the mechanism of on-site Coulomb interaction. There has been a great deal of interest in the Hubbard model due to its relevance to frontier problems in condensed matter physics such as high-temperature superconductivity and the trapping of ultra-cold atoms in optical lattices. While the Hubbard model is much more challenging and sophisticated than the tight-binding model, it can serve as a paradigm to gain significant physical insights into many-body relativistic quantum manifestations of distinct type of classical dynamics.

In our research, we focused on graphene systems and studied the particular phenomenon of quantum resonant tunneling. The typical setting of a quantum tunneling system consists of two symmetric potential wells separated by a potential barrier in between. The whole system, which includes the left and right wells as well as the barrier, is closed, and its geometrical shape can be chosen to yield characteristically distinct types of dynamics in the classical limit.

In order to uncover the unique relativistic quantum phenomena caused by classical chaos in the presence of many-body interactions, we first studied the class of integrable systems of rectangular shape. Since the whole system is closed, we calculated the eigenenergies and investigated various eigenstates from the

mean-field Hubbard Hamiltonian. A striking finding is the emergence of a class of eigenstates with near zero tunneling. In particular, for such an eigenstate, the spin-up and spin-down wavefunctions are typically separated, i.e., the spin-up electrons reside in only one potential well while the spin-down electrons reside in the other, as exemplified in Fig. 6. As a result, if the initial state is spin-up in one potential well, it is localized and will stay in the same well practically for an infinite amount of time with little quantum tunneling. When the potential term describing the on-site Coulomb interactions is removed so that the Hamiltonian becomes that of the tight-binding type, such localized states no longer exist, indicating that they are result of electron-electron interactions and consequently a distinct many-body phenomenon. We derived an approximate theory, based on the simplified picture of one-dimensional tunneling of massless Dirac fermions, to explain the physical origin of the localized states. We further found that, when the geometrical shape is that of stadium or bowtie so that the classical dynamics is chaotic, the localized states are greatly suppressed and the tunneling rates are significant. This means that, classical chaos is capable of effectively removing the localized states. In addition to the classically integrable and fully chaotic domains, we also considered a class of domains, the mushroom billiard, in which the classical dynamics is mixed or nonhyperbolic with coexisting regular and chaotic components in the phase space. We found that, due to the chaotic component, quantum tunneling can also be regularized and enhanced.

From the standpoint of device application, the localized states present an obstacle to effective tunneling and such states are therefore undesirable. The main message is then that chaos can significantly enhance the tunneling process in realistic situations where electron-electron interactions are present. This implies that classical chaos is capable of facilitating greatly relativistic quantum tunneling, which is desirable in the development of nanoscale devices such as graphene-based resonant-tunneling diodes.

Details of this work can be found in

- L. Ying, G.-L. Wang, L. Huang, and Y.-C. Lai, “Quantum chaotic tunneling in graphene systems with electron-electron interactions,” *Physical Review B* **90**, 224301, 1-14 (2014).

3.2 Conductance fluctuations associated with quantum transport in graphene systems

A fundamental problem in quantum transport through nanoscale devices is conductance fluctuations. Consider, for example, a quantum-dot system. As the Fermi energy of the conducting electrons is varied, the conductance can exhibit fluctuations of distinct characteristics, depending on the geometrical shape of the dot. Research in the past two decades demonstrated that the nature of the corresponding classical dynamics can play a key role in the conductance-fluctuation pattern. For example, when the classical scattering dynamics is integrable or has a mixed phase-space structure, there can be sharp fluctuations in the conductance curve. However, when the classical dynamics is fully chaotic, the conductance variations tend to be more smooth.

Qualitatively, why the conductance-fluctuation patterns depend on the nature of the corresponding classical dynamics can be understood, as follows. Sharp conductance fluctuations are caused by quantum pointer states, which are resonant states of finite but long lifetime formed inside the nanostructure, a kind of Fano resonance. For example, for a quantum-dot system whose classical dynamics is regular or contains a significantly regular component, there are stable periodic orbits. For the corresponding closed dot geometry, highly localized states can form about the periodic orbits, forming quantum scars. When leads are attached to the quantum dot so that the system becomes open, some periodic orbits can still survive, leading to resonant states, or quantum pointer states. Since the corresponding classical orbits are stable, the resonant states can have long lifetime, so they couple to the leads only weakly. As a result, very narrow resonances can form about the energy values that are effectively the eigenenergies in the corresponding closed system. When a modification to the dot geometry is introduced so that the underlying classical dynamics becomes

fully chaotic, no stable periodic orbits can exist. While scars can still be formed around classically unstable periodic orbits in the closed system, the corresponding resonant states in the open system generally will have much shorter lifetimes, effectively eliminating the narrow resonances in the conductance fluctuation pattern. This understanding also led to the idea that the interplay between the classical dynamics and the conductance-fluctuation pattern can be exploited for effectively modulating quantum transport (Sec. 3.2.2).

3.2.1 Insensitivity of conductance fluctuations to the nature of classical dynamics

We studied conductance fluctuation patterns in graphene devices. In a graphene quantum dot, the characteristics of conductance fluctuations depend on the nature of the classical dynamics in a way similar to that for conventional two-dimensional semiconductor quantum dot systems. However, the magnetic properties of graphene are quite different from those associated with conventional two-dimensional electrical gas (2DEG) systems because of the special lattice structure of graphene and the relativistic quantum behavior of quasiparticles. For example, in graphene quantum Hall effect can be observed even at room temperature due to the massless Dirac fermion nature of quasiparticles and significantly reduced scattering effects. Especially, the linear energy-momentum relation in graphene stipulates that the Landau levels are distributed according to $\pm\sqrt{N}$, where N is the Landau index, versus the proportional dependence on N in conventional 2DEG systems.

Our primary goal was to uncover any unique characteristics of conductance fluctuations in presence of a magnetic field, in the framework of non-equilibrium Green's function (NEGF). Our investigation revealed two surprising phenomena. First, in the parameter plane spanned by the strength of a perpendicular magnetic field and the Fermi energy, there are regions of regular and random conductance oscillations. As the Fermi energy or the magnetic field strength is changed, the fluctuations can be quite regular and then random, and vice versa, implying a kind of "coexistence" of regular and irregular conductance fluctuations as a single physical parameter is varied. Second, there exists a universal scaling behavior characterizing the magnetic conductance-fluctuation patterns, which does not depend on the nature of the classical dynamics. Through examining the edge states between adjacent Landau levels and the correspondence of their formation to conductance resonances, we found that, as the energy is increased or the magnetic-field strength is decreased so that a Landau level is crossed, a new set of fluctuation patterns emerges, leading to nearly periodic oscillations but of different period in the conductance versus the Fermi energy or the magnetic-field strength. Irregularity associated with the conductance fluctuations is developed as more periodic patterns set in. Considering that previous works pointed to the critical role played by the nature of classical dynamics in conductance fluctuations, our finding that the presence of magnetic field can greatly suppress this sensitivity to classical dynamics was striking.

One pertinent question, when a perpendicular magnetic field is applied to a quantum dot, is how conductance fluctuations are affected by the size of the quantum dot. In a previous experimental study of quantum dots of size ranging from $0.7\ \mu\text{m}$ to $1.2\ \mu\text{m}$, nearly periodic conductance oscillations had been found as the magnetic-field strength is varied. The frequency of the oscillation pattern, the so-called magnetic frequency, follows a scaling relation with the edge size of the dot. In a study of the magnetic scaling behavior for graphene quantum dots, it was found that for small dots of edge size less than $0.3\ \mu\text{m}$, the magnetic frequency exhibits a scaling relation with the dot area. We focused on a particular set of scarred orbits and examined the resulting conductance oscillations. We found, for graphene quantum dots, below the first Landau level, the conductance exhibits periodic oscillations with the magnetic-field strength and with the Fermi energy. The magnetic frequency scales linearly with the dot size. However, the energy frequency, the inverse of the variation in the Fermi energy for the conductance to complete one cycle of oscillation, scales inversely with the dot size. Beyond the regime of periodic conductance oscillations, new sets of scarred orbits emerge and evolve, each with its own period, leading to random-like conductance fluctuations. The

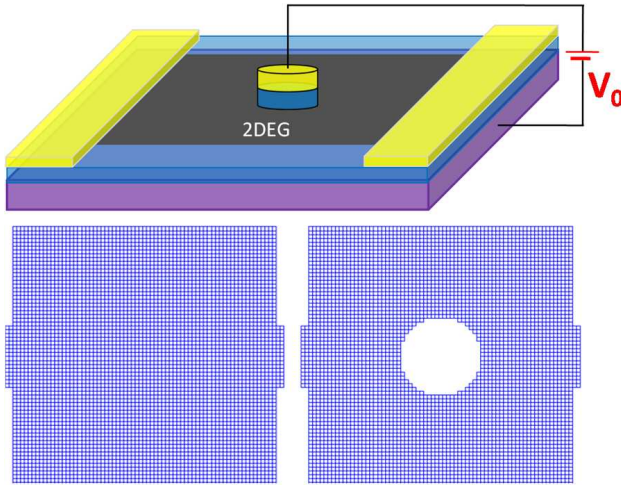


Figure 7: A possible experimental scheme to harness transport through a semiconductor 2DEG quantum-dot system, where 2DEG is formed at the GaAs/Al_{0.3}Ga_{0.7}As hetero-interface. The heterostructure sits on a n^+ Si substrate (purple), covered by 300 nm SiO₂ (blue) and contacted by Au/Cr (yellow). When a suitable gate voltage is applied to generate a *circular forbidden region* (for classical orbits) at the center of the device, the resulting closed system is a Sinai billiard. Open quantum-dot system can be formed by attaching leads to the billiard system. In our study, we placed the leads in the middle of the dot, as shown in the second row. Similar idea can be applied to graphene to implement the chaos-based harnessing of quantum transport.

remarkable feature is that these scaling behaviors are independent of the nature of the underlying classical dynamics, i.e., regular or chaotic.

Details of this work can be found in

- L. Ying, L. Huang, Y.-C. Lai, and C. Grebogi, “Conductance fluctuations in graphene systems: The relevance of classical dynamics,” *Physical Review B* **85**, 245448, 1-7 (2012).

3.2.2 Harnessing quantum transport by transient chaos

Controlling chaos in dynamical systems has been studied for more than two decades since the seminal work of Ott, Grebogi, and Yorke in 1990. The basic idea was that chaos, while signifying random or irregular behavior, should not be viewed as a nuisance in applications of nonlinear dynamical systems. In fact, given a chaotic system, that there are an infinite number of unstable periodic orbits embedded in the underlying chaotic invariant set means that there are an equally infinite number of choices for the operational state of the system depending on need, provided that any such state can be stabilized. Then, the intrinsically sensitive dependence on initial conditions, the hallmark of any chaotic system, implies that it is possible to apply small perturbations to stabilize the system about any desirable state. Controlling chaos has since been studied extensively and examples of successful experimental implementation abound in physical, chemical, biological, and engineering systems. The vast literature on controlling chaos, however, was limited to nonlinear dynamical systems in the classical domain. We elaborated the idea that chaos may be exploited to harness or modulate quantum-mechanical behaviors by conducting a systematic study of the role of transient chaos in modulating quantum transport dynamics. In particular, in applications such as the development of electronic circuits and nanoscale sensors, severe conductance fluctuations are undesirable and are to be eliminated so that stable device operation can be achieved. An outstanding question was then, can practical and experimentally feasible schemes be articulated to modulate the quantum conductance fluctuations? We demonstrated and provided theoretical understanding that classical *transient chaos* can be used to effectively modulate conductance-fluctuation patterns associated with quantum transport through nanostructures.

A quantum dot typically consists of a finite device region of certain geometrical shape, such as a square, a circle, or a stadium, and a number of leads connected with the device region. To demonstrate quantum

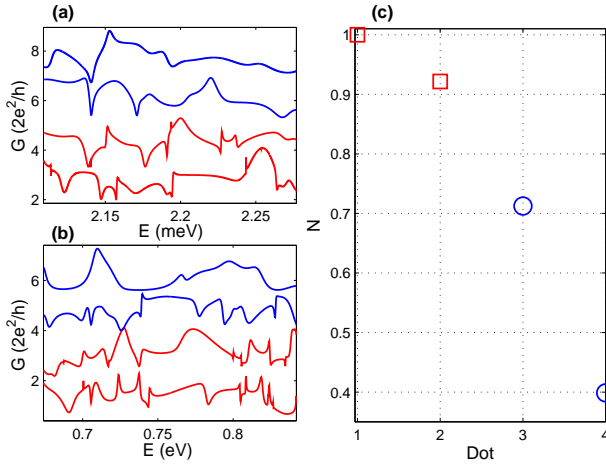


Figure 8: Exploiting chaos for quantum control. (a) Conductance versus Fermi energy for four semiconductor 2DEG quantum-dot systems (bottom to top): rectangular dot, rectangular dot with a rectangular forbidden region of area $0.25 \mu\text{m} \times 0.25 \mu\text{m}$, and Sinai dots of radii $R=0.14 \mu\text{m}$ and $R=0.28 \mu\text{m}$. The area of the original rectangular dot is $1 \mu\text{m} \times 1 \mu\text{m}$ and the lead width is $0.22 \mu\text{m}$. (b) Conductance fluctuation patterns for graphene quantum dots of the same geometry as in (a). The curves in (a) and (b) were shifted vertically by some arbitrary values to facilitate visualization. (c) Normalized number of *sharp resonances*, which is defined to quantify the degree of transmission fluctuations, corresponding to the four cases in (a), i.e., the top two rectangular symbols represent two rectangular quantum dots while the bottom two circle symbols correspond to two Sinai quantum dots.

harnessing by using chaos, we conceived generating a region about the center of the device or structure with high potential so that it is impenetrable to classical particles. For example, consider a rectangular quantum dot. When it is closed, the corresponding classical dynamics is integrable so that extremely narrow resonances can arise in the quantum transport dynamics of the open-dot system. Now imagine applying a gate voltage to generate a circular, classically forbidden region about the center of the dot, as shown schematically in Fig. 7. In general, the potential profile will be smooth in space. However, the scattering behavior is qualitatively similar to that from an infinite potential well. Thus in our simulation we adopted the infinite potential-well assumption for the central region, which defines a “forbidden” region. Varying the voltage V_0 can change the effective radius R of the forbidden region. Classically, the closed system is thus a Sinai billiard, which is fully chaotic, insofar as the radius of the central potential region R is not zero. When leads are connected to the device region so as to open the system, chaos becomes transient. The dynamical characteristics of the underlying chaotic invariant set can be adjusted in a continuous manner by increasing the radius R . Quantum mechanically we thus expect to observe increasingly smooth variations in the conductance with, e.g., the Fermi energy, which we demonstrated using both semiconductor 2DEG and graphene systems, as shown in Fig. 8.

Details of this work can be found in

- R. Yang, L. Huang, Y.-C. Lai, C. Grebogi, and L. M. Pecora, “Harnessing quantum transport by transient chaos,” *Chaos* **23**, 013125, 1-9 (2013).

3.2.3 Quantum chaotic scattering in graphene systems without invariant classical dynamics

In open Hamiltonian systems, a fundamental type of chaotic behaviors is transient chaos, which leads to chaotic scattering. Quantum chaotic scattering, referred to as the study of the unique quantum behaviors caused by chaotic scattering in the corresponding classical system, is highly relevant to a number of fields in physics, such as atomic physics, condensed matter physics, and acoustics.

The fundamental quantity characterizing a quantum scattering system is the scattering matrix, or the

S-matrix, whose elements are the transition probabilities between quantum states of the system before and after the scattering. The formulation of the S-matrix in terms of classical quantities had been of great interest in chemical physics even before chaos started attracting wide attention. The seminal contribution by Miller in 1975, who obtained a formula for S-matrix elements in terms of purely classical quantities in the semiclassical regime for reactive scattering systems, became the fundamental tool in the study of quantum chaotic scattering. Given a system that exhibits chaotic scattering in the classical limit, the S-matrix elements in the semiclassical regime exhibit random fluctuations as some physical parameters of the system, such as the energy of the scattering particle or the strength of some externally applied magnetic field, change in a classically small but quantum-mechanically large range. Depending on whether classical scattering is hyperbolic (fully chaotic) or nonhyperbolic (mixed), the statistical properties of the fluctuations in the S-matrix elements can be quite distinct. In particular, the fluctuation patterns can be characterized by the energy-correlation function, where a faster decay of the function from unity as the energy difference is increased points to more severe fluctuation patterns. In their seminal work in 1988, Blümel and Smilansky showed that the energy-correlation function is related to the decay law of trajectories in the classical phase space as a Fourier-transform pair. Thus, if classical trajectories decay faster from the scattering region, the energy-correlation function decreases more slowly from unity, and vice versa.

A tacit assumption employed in previous works on quantum chaotic scattering is that, as a physical parameter changes, the classical dynamics is invariant so that the decay law remains unchanged, rendering meaningful its Fourier transform. Such situations arise in physical systems, for example, transport of electrons through a quantum dot in the absence of magnetic field where the corresponding classical dynamics is essentially that of an open billiard and so does not depend on the electron energy. However, when a magnetic field is present, the characteristics of classical dynamics can change drastically with the particle energy. In particular, in an early work, Breymann, Kovács and Tél studied chaotic scattering of charged particles in an open three-disk billiard subject to a perpendicular magnetic field, which is effectively a three-terminal quantum-dot system. Classically, since the Lorentz force depends on both the particle energy and the magnetic field, even for a fixed magnetic field strength the dynamics will depend on the energy. We thus faced a challenge that, as the particle energy is systematically changed, there is no unique classical correspondence. Quantum mechanically, the scattering matrix elements will exhibit fluctuations with the particle energy, but the Fourier-transform formula relating the energy autocorrelation function and classical particle decay law is no longer applicable. How could then the fluctuation patterns be characterized and understood?

To investigate quantum chaotic scattering in absence of invariant classical dynamics, we used quantum-transport systems in two-dimensions in the presence of a perpendicular magnetic field as a paradigm. In particular, utilizing multi-terminal graphene quantum dots as a prototypical class of systems, we articulated that quantum chaotic scattering can be physically characterized by the complex eigenvalues of the non-Hermitian Hamiltonian of the underlying system. A quantum-dot system can in general be decomposed into two parts: a closed device or scattering region and the set of semi-infinite electronic waveguides (or leads). The device Hamiltonian is Hermitian and permits a discrete set of eigenvalues, but the leads possess a continuum of energy spectrum, mathematically described by complex self-energy terms. Adding the self-energies to the device Hamiltonian leads to a non-Hermitian Hamiltonian describing the entire open quantum system, whose eigenvalues are in general complex. The key physics to quantum chaotic scattering is that the inverse of the imaginary part of a complex eigenvalue is nothing but the lifetime of the corresponding eigenstate in the scattering region. Thus, when an energy eigenvalue possesses an extremely small imaginary part, the corresponding eigenstate has a long lifetime, which is effectively a strongly localized state. The coupling between such a localized state and the leads, or the quantum “environment” of the device, must necessarily be weak. This leads to an abrupt change in the transmission or conductance over an extremely small energy scale, the so-called Fano resonance. Overall, in order to explain a numerically calculated or experimentally observed conductance-fluctuation pattern in quantum-dot systems in presence of a magnetic

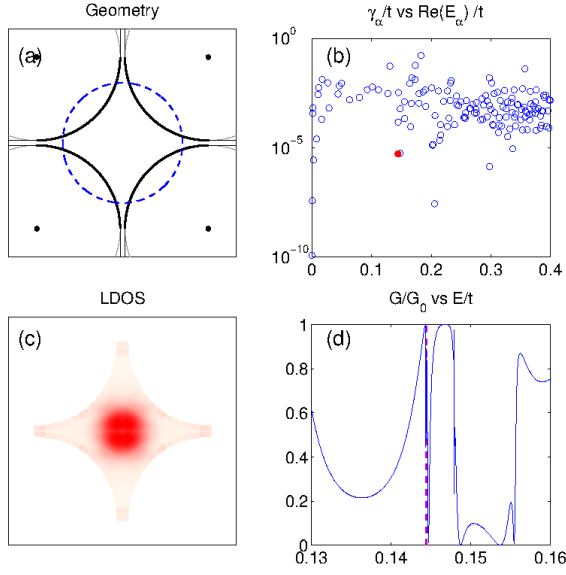


Figure 9: **Conductance fluctuations and characterization in a four-terminal graphene quantum-dot system under a perpendicular magnetic field:** (a) geometry of the system, where the scattering (device) region is denoted by thick black curves and the region in which the perpendicular magnetic field is applied is represented by the blue dashed circle; (b) values of the eigenenergies in the complex plane, where eigenvalues with extremely small imaginary part begin to appear after the first Landau level arises; (c) a representative strongly localized state with weak coupling to the leads, corresponding to the filled red circle in (b); (d) the corresponding local conductance-fluctuation pattern, where the vertical red dashed line corresponds to the real energy value of the filled red circle in (b).

field, one can calculate the complex eigenenergies of the corresponding non-Hermitian Hamiltonian. Energy regions in which the eigenenergies exhibit small imaginary parts are the regions where severe conductance fluctuations can be anticipated. These results should provide insights into the development of graphene-based quantum transport systems.

Some representative results are shown in Fig. 9.

Details of this work can be found in

- G.-L. Wang, L. Ying, Y.-C. Lai, and C. Grebogi, “Quantum chaotic scattering in graphene systems in the absence of invariant classical dynamics,” *Physical Review E* **87**, 052908, 1-9 (2013).

3.2.4 Universal transform for Fano resonance

A fundamental phenomenon associated with quantum or wave scattering dynamics is Fano resonance. A typical scattering system consists of incoming channels, a scatterer or a conductor, and outgoing channels. The scatterer, when isolated, can be regarded as a closed system with a discrete spectrum of intrinsic energy levels. When the energy of the incoming particle or wave matches an energy level, a resonant behavior can arise in some experimentally measurable quantities, such as the conductance in a quantum-transport system. The resonance profile is typically asymmetric, and can in general be expressed as $(\epsilon + q)^2 / (\epsilon^2 + 1)$, where ϵ is the normalized energy deviation from the center of the resonance, and q is the parameter characterizing the degree of asymmetry of the resonance. The asymmetry parameter q is of great experimental importance as it determines how experimental data can be fitted by the Fano profile.

The profile was first derived by Fano in the study of inelastic scattering of electrons off the helium atom and auto-ionization, although the phenomenon was predicted earlier in elastic neutron scattering. Being a general wave interference phenomenon, Fano resonance has been found in many contexts in physics, such as photon-ionization, Raman scattering, photon-absorption in quantum-well structure, scanning microscopy tunneling in the presence of impurity, transport through a single-electron transistor and through the Aharonov-Bohm interferometer, transport through crossed carbon nanotubes, microwave scattering, plasmonic nanostructures and metamaterials, and optical resonances. Applications exploiting Fano resonance have even been proposed for biochemical sensors.

An unsettled and somewhat controversial issue was whether the asymmetric parameter q should be real or complex. The issue is of high experimental relevance. For example, it was demonstrated that, in single-channel scattering quantum-transport systems, q is strictly real if the time-reversal symmetry (TRS) is preserved. When the TRS is broken, q will generally be complex, which was observed experimentally in an Aharonov-Bohm interferometer with a quantum dot embedded in one of the arms. With a changing magnetic field, q oscillates, which was explained but again by using the single-channel scattering model. Later, it was shown that for multi-channel scattering, q is in general complex even when TRS is not broken. It was also proposed that complex q parameter can be used to characterize the dephasing time. The variation in q and the degree of decoherence were studied in microwave billiards where a non-zero imaginary part of q was declared. Moreover, it was suggested that the trajectory of q in its complex plane could be used to probe whether decoherence is caused by dissipation or dephasing. Complex q parameter was used in other contexts as well, such as the second-order effects in helium auto-ionization excited by electron impact and ultrashort laser excitation in the bismuth single crystal.

During the project period, we established the universality of the asymmetric Fano resonance profile and addressed the question of whether the fundamental q parameter should be complex or real. To be concrete, we focused on low-dimensional quantum-transport systems exemplified by quantum dots or quantum point contacts. Despite the relatively long history of research on Fano resonance, for quantum transport systems the asymmetric profile had been studied much later. Our approach to probing into the nature of the q parameter consisted of two steps. First, by using the non-equilibrium Green's function to calculate, for all scattering channels, the transmission as a function of the Fermi energy, we derived a formula for the resonance profile and verified it numerically using graphene quantum dot systems. The key approximation employed is that the self-energy terms and the coupling functions are slow variables, so for energy near an isolated resonance, the Green's function can be decomposed into a fast and a slowly varying components. The formula also leads to an expression of the width of the resonance. Second, we showed that our formula bridges naturally the conventional Fano formulas with complex and real q parameters. In particular, we found a simple mathematical transform that can convert the Fano formula with complex q parameter to our formula, and another transform that turns our formula into the Fano formula with real q parameter. The implication is that, for any experimental situation the conventional Fano formulas with complex or real q parameter are equally applicable, given that an offset can be added to the real q Fano formula. In our analysis, no detailed information about the specific system is required. Thus, our formula and its direct consequence hold for any coherent transport dynamics for both bosons and fermions. Especially, for bosons, i.e., phonons, the control parameter is not the energy but the frequency, thus requiring only a straightforward modification in our formula. For fermion transport, our formula is valid for small-scale electronic devices such as quantum-dot systems, quantum point contacts, nanoscale heterostructures, and single-molecule transport devices.

Details of this work can be found in

- L. Huang, Y.-C. Lai, H.-G. Luo, and C. Grebogi, "Universal formalism of Fano resonance," *AIP Advances* **5**, 017137, 1-18 (2015).

3.3 Optical branched waves, optomechanical systems, and fluid turbulence

3.3.1 Emergence of scaling associated with complex branched wave structures in optical medium

When waves propagate through random media, extreme events and complex structures such as rogue waves and branched, fractal-like wave patterns can form. There is a great interest in complex wave phenomena due to their occurrences in a host of physical systems. For example, in oceanography, rogue waves are an issue of great concern. In the past fifteen years or so there had been experimental and theoretical studies of rogue

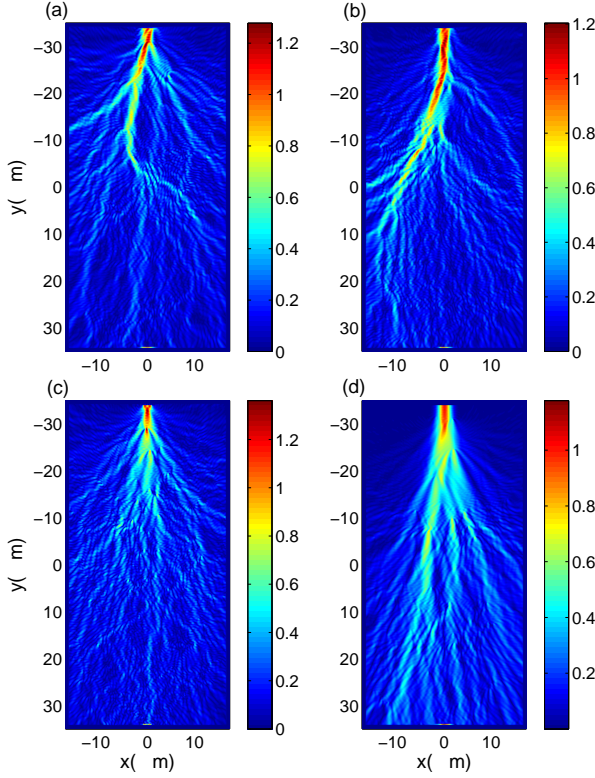


Figure 10: Fractal-like branched wave patterns in optical media. For a rectangular optical medium of size $35\mu\text{m} \times 70\mu\text{m}$ with $N = 300$ spatially localized, Gaussian-shaped scatterers, typical spatial distributions of the magnetic field strength $|H|$ of the scattered waves. In each case, an external polarized, monochromatic, Gaussian wave of width $w = 1\mu\text{m}$ and unit intensity is sent from the top of the region. The spatial distribution function of the refractive index for the whole system is $n(\mathbf{r}) = n_0 + \sum_{i=1}^N \Delta n_i \exp[-|\mathbf{r} - \mathbf{r}_i|^2 / (2\sigma^2)]$, where Δn_i is the magnitude of the refractive index of the i th scatterer relative to that of the medium, and $\mathbf{r} = (x, y)$ is a two-dimensional vector. Each scatterer is characterized by a Gaussian-shaped refractive index profile, whose effective radius is σ . For panels (a) and (b), the refractive-index variations are negative: $\Delta n_i = -0.5$. The variation is positive for panel (c): $\Delta n_i = 0.5$. For panel (d), Δn_i is randomly selected from the range $[-0.5, 0.5]$. Other parameters are the same for all panels: $n_0 = 1$, $\lambda = 1\mu\text{m}$, and $\sigma = 0.22$.

waves arising from long-range acoustic wave propagation through ocean's sound channel, as well as large-scale experiments on directional ocean waves to probe the physical and dynamical origin of these extreme waves. Extreme events and complex wave patterns were also identified in many other physical situations such as light propagation in doped fibers, acoustic turbulence in superfluid helium, resonances in nonlinear optical cavities, linear light-wave propagation in multi-mode glass fiber, and electronic transport in semiconductor two-dimensional electron gas (2DEG) systems. Despite previous efforts, an accepted, relatively complete understanding of complex extreme waves at the level of fundamental physics was lacking.

To illustrate the extent to which complex branched wave patterns were previously understood, we choose electronic transport in 2DEG systems as an example. It was found by a Harvard group in 2001 that electron flows from a quantum point contact exhibit a branched or fractal-like behavior with highly non-uniform amplitude distribution in the physical space. The observed separated, narrow strands of greatly enhanced electron wave intensities were argued to be caused by random background potentials and quantum coherent phase interference among the electronic wave functions. Subsequently a theory was proposed to predict the statistical distribution of the intensities of branched electron flows in the presence of weak, correlated Gaussian random potentials.

The generic origin of wave branching behavior has been a matter of active debate. A tacit assumption in most previous investigations was nonlinearity in the underlying medium. In particular, it had been believed that the existence of many uncorrelated, spatially randomly distributed wave elements is key to the occurrence of these exotic wave patterns. These elements can be, for example, solitons in nonlinear systems. However, it was demonstrated experimentally in a microwave system and in a multi-mode optical fiber that branched wave patterns can occur even in the absence of nonlinearity. In fact, in the latter case, granularity of light speckles at the fiber exit and inhomogeneity in the spatial clustering of the speckle patterns are speculated to be the two ingredients that trigger complex wave patterns. These works thus demonstrated

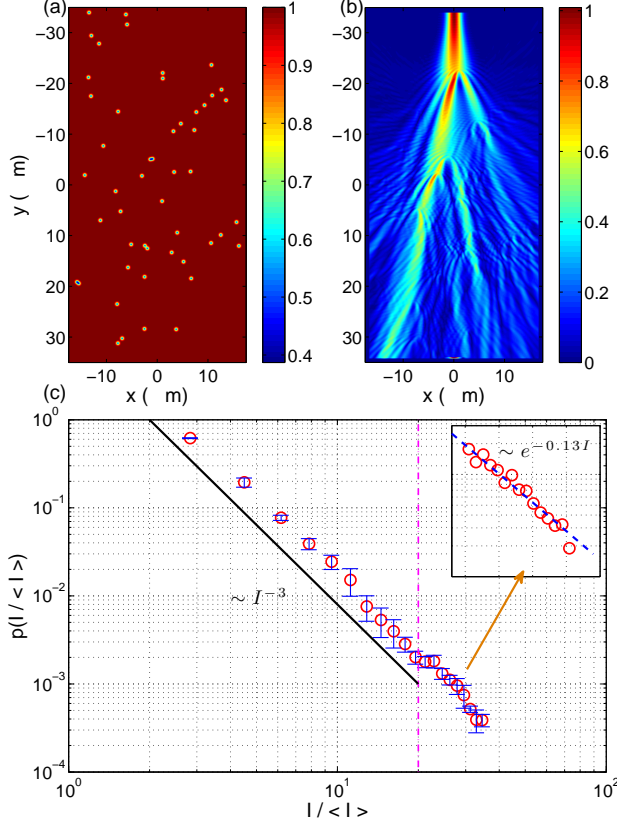


Figure 11: Intensity scaling of fractal-like, branched wave patterns in optical media. (a) Distribution of 50 random scatterers in the $35\mu\text{m} \times 70\mu\text{m}$ uniform medium. (b) Branched wave magnitude from FDFD simulation. The incident wave is uniform with width λ and sent in the $+y$ direction, where $\lambda = 1\mu\text{m}$, $n_0 = 1$, $\Delta n = -0.5$, $\sigma = 0.23\mu\text{m}$. (c) Numerically obtained scaling law of the intensity distribution (blue circles) on a logarithmic scale and the theoretical prediction (solid line $\sim I^{-3}$). There is an exponential cut-off in the high-intensity regime, as shown in the inset.

that nonlinearity is not absolutely essential for the emergence of these extreme waves. An issue of significant theoretical and experimental interest concerns thus a minimal, physical model that can generate robust branched wave patterns, so that their generic and physical origin may be elucidated. A related issue is the statistical properties of these waves. In this regard, a general observation in all contexts where branched wave structures arise is the non-Gaussian statistics of the wave amplitude. Typically there is a long tail in the probability density function, which characterizes the extreme intensity of the waves. An essential requirement for a valid minimal model of branched wave patterns is thus that it generate the universally observed long-tail distribution in the wave intensity.

During the project period, we proposed a class of minimal models for branched wave patterns in the context of wave propagation in two-dimensional optical medium. The model contains two basic physical elements: (1) a uniform medium of finite size and (2) spatially localized scatterers randomly distributed in the medium, the refractive indices of which deviate from that of the background medium. The deviations can occur in both ways which, in the case of negative deviation, may correspond to scatterers that are effectively negative-indexed, or *metamaterials*. The second element is required for generating dynamics beyond simple linear wave propagation. We demonstrated that such a minimal model can generate robust branched wave patterns, regardless of the detailed distribution of the refractive-index deviations associated with the random scatterers. We then developed a comprehensive analytic treatment of the minimal model by focusing on the theoretical derivation of the power-law type of long-tail type of distribution in the wave intensities.

More specifically, we considered the setting where a polarized monochromatic light propagates in a dielectric optical medium with random structural imperfections characterized by random refractive-index disorders (scatterers) of size comparable to the wavelength. We then employed the standard finite-difference frequency-domain (FDFD) method to calculate the intensity of the scattered field through multiple scatterers.

In the weak scattering limit, i.e., when the wavelength λ is much smaller than the mean free path l , we obtained striking branching flow structures of propagating light, similar to those observed in the 2DEG and microwave transport experiments. Examples of branched wave patterns are shown in Fig. 10. As the spatial density of the scatterers is increased, the intensity patterns exhibit more pronounced fractal-like behavior, where branches of extraordinarily high intensities tend to enhance themselves when forking into narrower and even smaller paths. Anderson localization of light is also observed as the mean free path approaches the strong scattering limit ($l \sim \lambda$). Our extensive numerical computation confirmed that the branched structure can result from the caustics of the flow rather than the valleys of the random scatterers. We found that branched waves generically arise in the regime between weak scattering and strong localization of light waves.

In order to obtain a comprehensive understanding of the occurrence of branched waves in optical media and also to uncover their statistics, we developed a detailed analytic theory. Utilizing the Green's function method, we treated the scattering of two-dimensional polarized light wave off a single scatterer and obtained a theoretical explanation for the reason why the wavelength needs to be comparable to the size of the scatterer in order for noticeable large fluctuations of branching strands to be observed. In contrast to the existing theory which deals with stretches and folds in classical ray dynamics in a concrete and somewhat abstract manner, our theory enables us to *visualize* the branching flow structures of the scattered light intensities in different angular directions. Based on the results from scattering off a single scatterer, we next extended our treatment to multiple scatterers. In this case, coherent backscattering and recurrent multiple scattering become important, and they together contribute dominantly to the formation of extremely large amplitude events. This unstable branch stretching and accumulation process is highly sensitive to the scatterers' spatial distribution and thus is critical to the formation of fractal-like wave patterns. Because of the large intensity fluctuations caused by wave interference and the complexity of random-scatterer configuration, it is essential to focus on the statistical distribution function of light intensities. We succeeded in deriving a formula for the distribution function of high intensities, which follows an algebraic scaling law in the weak-scattering limit. An example of the algebraic intensity distribution is shown in Fig. 11. The physical significance of the algebraic scaling means that, associated with the branched waves, there are points in the space at which exceedingly large intensities can arise, the "hot" spots, in contrast to situations governed by Gaussian type of intensity distributions.

Details of this work can be found in

- X. Ni, Y.-C. Lai, and W.-X. Wang, "Emergence of scaling associated with complex branched wave structures in optical medium," *Chaos* **22**, 043116, 1-11 (2012).

3.3.2 Enhancement of quantum entanglement by nonlinear dynamics in optomechanical systems

When a quantum system contains several degrees of freedom, entanglement among them can arise. Quantum entanglement is a form of quantum superposition, which can be best understood by considering a pair of particles such as electrons. When two such particles are entangled, each particle cannot be fully described without considering the other because they both belong to a single quantum superposition state. Without external disturbance, e.g., a measurement, the particles remain in the entangled state. In order to determine the value of some physical quantity associated with one particle in the entangled state, it is necessary to apply perturbation to the system, after which the state of the particle takes on a definite value. However, the other particle in the originally entangled pair will have the corresponding correlated value for any subsequent time, regardless of the physical distance between the two particles. The remarkable phenomenon of quantum entanglement has been observed in experiments of large molecules and even small diamonds, and it is the foundation of modern quantum computing and information science.

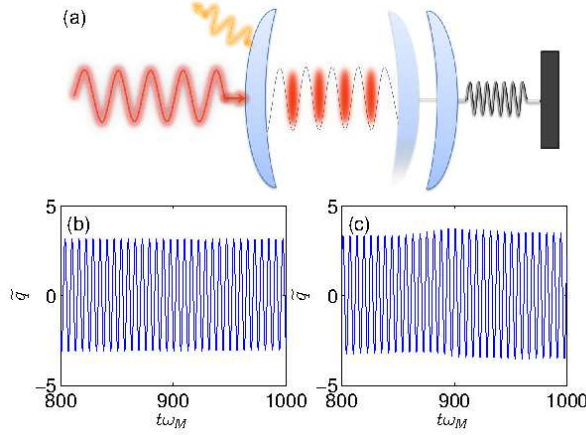


Figure 12: Nonlinear dynamics in optomechanical systems. (a) Schematic illustration of a typical optomechanical system driven by a single-mode laser. The laser beam enters the optical cavity through a fixed mirror but photons inside the cavity can also decay through the mirror. The movable mirror is totally reflecting and linked to a mechanical oscillator. (b,c) Time series of the dimensionless canonical coordinate \tilde{q} for situations where the classical dynamics are periodic and quasiperiodic, respectively.

While the principle of superposition is defined with respect to linear systems, our physical world, when viewed classically, is nonlinear. A nonlinear dynamical system can exhibit all kinds of interesting phenomena such as periodic oscillations, quasiperiodicity and chaos, and they can have distinct fingerprints or manifestations when the system is treated quantum-mechanically. Especially, the studies of quantum manifestations of classical chaos constitute the field of quantum chaos, and there have been tremendous efforts in the past four decades in this field. However, there is no classical correspondence of quantum entanglement in the strict sense, as the measurement of a physical state at one place immediately influences the measurement at the other. In view of the ubiquity of nonlinear dynamics in physical systems and the fundamental importance of quantum entanglement, curiosity demands that we ask the following question: What is the interplay between nonlinear dynamics and quantum entanglement? Or more explicitly, when the corresponding classical system is nonlinear, what are the signatures on quantum entanglement? This issue may have significant practical implications in developing devices and systems for quantum computing and quantum information processing.

During the project period, we addressed the nonlinear-dynamics/quantum-entanglement issue by using optomechanical systems, a field of intense recent investigation. As shown in Fig. 12, an optomechanical system consists of an optical cavity and a nanoscale mechanical oscillator, such as a cantilever. When a laser beam is introduced into the cavity, a resonant optical field is established that exerts a radiation force on the mechanical cantilever, causing it to oscillate. The mechanical oscillations in turn modulate the length of the optical cavity, hence its resonant frequency. There is thus coupling between the optical and the mechanical degrees of freedom. This coupling, or interaction, can lead to cooling of the mechanical oscillator toward the quantum ground state, a topic of great recent interest. The optomechanical coupling thus provides a straightforward way to entangle the optical with the mechanical modes. Previous works on quantum entanglement in optomechanical systems focused on the situations where the classical dynamics are either steady-state or periodic oscillations. Our goal was to search for nonlinear dynamical behaviors beyond steady-state solutions and periodic oscillations, and to investigate the effects of such behaviors on quantum entanglement.

Our main findings were the following. In an experimentally realizable parameter regime, as the power of the driving laser is increased, there is a transition from periodic to quasiperiodic motions, where in the latter, the system is strongly nonlinear with two incommensurate frequencies. Entanglement is enhanced towards the transition, vanishes as the transition point is reached, but is restored abruptly after the transition and continues to be enhanced as the system evolves deeply into the quasiperiodic regime. Representative results are shown in Fig. 13. A surprising result is that, with respect to time evolution, there is a direct correspondence of quantum entanglement to classical nonlinear dynamics. In particular, for classically periodic

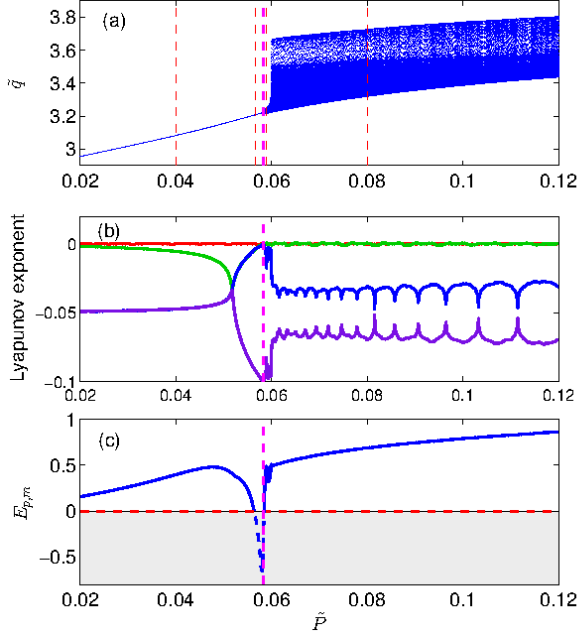


Figure 13: Enhancement of quantum entanglement by nonlinear dynamics in optomechanical systems. (a) Typical bifurcation diagram of the generic optomechanical system in Fig. 12(a), where the asymptotic values of the dimensionless canonical coordinate \tilde{q} are plotted versus the normalized external laser power \tilde{P} . The purple dashed line indicates the threshold driving power, at which a transition from periodic to quasiperiodic motions occur. The cavity length is $l_0 = 25\text{mm}$ with finesse $F = 6 \times 10^4$. (b) The entire spectrum of the Lyapunov exponents versus \tilde{P} , and (c) the maximum of the pseudo-entanglement measure \tilde{E}_p versus \tilde{P} .

dynamics, the time evolution of the entanglement measure is also periodic, but when the classical system enters into the quasiperiodic regime, the quantum entanglement measure exhibits a beats-like behavior with two distinct frequencies. The entanglement, especially when the classical system is quasiperiodic, is robust with respect to temperature variations. Thus, from the perspective of entanglement, there is a clear quantum manifestation and signature of nonlinear dynamics in optomechanical systems.

Details of this work can be found in

- G.-L. Wang, L. Huang, Y.-C. Lai, and C. Grebogi, “Nonlinear dynamics and quantum entanglement in optomechanical systems,” *Physical Review Letters* **112**, 110406, 1-6 (2014).

3.3.3 Low Reynolds-number turbulence in ferrofluids

A basic understanding of fluid turbulence is that it typically occurs at high values of the Reynolds number, a dimensionless quantity defined as $Re \equiv lu/v$, where l and u are the characteristic length scale and a typical velocity of the flow, respectively, and v is the kinematic viscosity. Take, for example, the paradigmatic setting of a uniform flow of velocity u flowing past a cylinder of diameter l . When Re is of the order of tens, the flow is regular. For Re in the hundreds, von Kármán vortex street forms behind the cylinder, breaking certain symmetries of the system. Fully developed turbulence, in which the broken symmetries are restored, occurs at very high Reynolds number, typically in the thousands. Because of the requirement of high Reynolds numbers, study of turbulence has historically been known as an extremely challenging problem, both experimentally where flow systems of enormous size and/or high velocity are required and computationally where unconventionally high resolution in the numerical integration of the Navier-Stokes equation is needed. It is thus desirable that fluid turbulence can emerge in physical flows of relatively low Reynolds numbers. At a fundamental level, such a flow system may present us with a new paradigm that can potentially yield significantly new insights into the emergence, characterization, mechanism, and control of fluid turbulence.

During the project period, we investigated the Taylor-Couette flow in finite systems (e.g., aspect ratio $\Gamma = 20$) where a rotating *ferrofluid* is confined by axial end walls, i.e., non-rotating lids, in the presence of

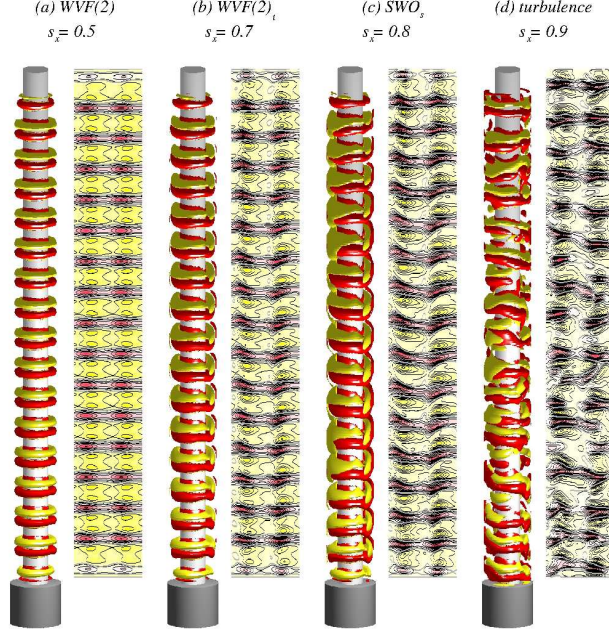


Figure 14: Transition to turbulence in a rotating ferrofluid. (a-d) For four values of the magnetic parameter (Niklas parameter) s_x corresponding to WVF₂ (wavy vortices with a two-fold symmetry), WVF_t (wavy vortices with a tilting pattern), SWO_p (standing waves with axial oscillations), and turbulence regimes, respectively, isosurfaces of azimuthal vorticity and contours of the radial velocity on an unrolled cylindrical surface in the annulus at midgap. Red (dark gray) and yellow (light gray) colors denote isosurfaces, and inflow and outflow for contour plots, respectively. The patterns in (c,d) are snapshots due to time dependence of the corresponding flow.

an external magnetic field. The classic Taylor-Couette flow of non-ferrofluid has been a computational and experimental paradigm to investigate a variety of nonlinear and complex dynamical phenomena, including transition to turbulence at high Reynolds numbers. The corresponding ferrofluid system we studied consists of two independently rotating, concentric cylinders with viscous ferrofluid filled in between, which has embedded within itself artificially dispersed magnetized nanoparticles. In absence of the external magnetic field, the magnetic moments of the nanoparticles are randomly oriented, leading to zero net magnetization for the entire fluid. In this case, the magnetized nanoparticles have little effect on the physical properties of the fluid such as density and viscosity. However, when a transverse magnetic field is applied, the physical properties of the fluid are significantly modified, leading to drastic changes in the underlying hydrodynamics. For example, for systems of rotating ferrofluid, an external magnetic field can stabilize regular dynamical states and induce dramatic changes in the flow topology. In general, the magnetic field can be used effectively as a control or bifurcation parameter of the system, whose change can lead to characteristically distinct types of hydrodynamical behaviors. Existing works on rotating ferrofluids however, were mostly concerned with steady *time-independent* flows. Time-dependent ferrofluid flows were investigated only recently but in the non-turbulent regime.

The interaction between ferrofluid and magnetic field leads to additional terms in the Navier-Stokes equation. Our extensive computations revealed a sequence of bifurcations leading to time-dependent flow solutions such as standing waves with periodic or quasiperiodic oscillations, and turbulence. Surprisingly, we found that turbulence can occur for Reynolds numbers at least one order of magnitude smaller than those required for turbulence to arise in conventional fluids. The occurrence of turbulence was ascertained by a bifurcation analysis and by examining the characteristics of the physical quantities such as the energy, the wave number, and the angular momentum. We also found that the onset of turbulence can be determined accurately, in contrast to classical fluid turbulence where such a determination is typically qualitative and involves a high degree of uncertainty. Our findings have the following implications: (1) ferrofluids under magnetic field is a new paradigm for investigating turbulence, especially experimentally where the study can be greatly facilitated due to the dramatic relaxation in the Reynolds-number requirement, (2) the critical magnetic-field strength for the onset of turbulence can be pinned down precisely, possibly leading to deeper insights into the physical and dynamical origins of the transition, and (3) turbulence can be controlled

externally, e.g., by an external magnetic field.

Details of this work can be found in

- S. Altmeyer, Y.-H. Do, and Y.-C. Lai, “Transition to turbulence in Taylor-Couette ferrofluidic flow,” *Nature Scientific Reports* **5**, 10781, 1-12 (2015).

3.4 Superpersistent currents and whispering gallery modes in relativistic quantum chaotic systems

A remarkable phenomenon in the quantum world is persistent currents (PCs), permanent currents without any external source, which are generated by the Aharonov-Bohm (AB) effect in non-superconducting systems. PCs have been observed experimentally in metallic and semiconductor rings in the mesoscopic regime. Theoretical efforts have been focused on the effects of bulk disorders, electron-electron interactions, spin-orbital interactions, and electromagnetic radiation on PCs, typically studied in the diffusive regime using idealized circular-symmetric rings and cylinders. Rapid advances in nanotechnology have made it feasible to fabricate mesoscopic devices with mean free path larger than their sizes at sufficiently low temperatures (the ballistic transport regime). The AB system can thus be modeled as a quantum ballistic billiard in which the particles are scattered at the boundaries of the domain. As a result, the boundary shape becomes highly relevant. In experiments, uncontrollable boundary imperfections are inevitable even when there are no bulk disorders. It is thus of interest to study the effects of boundaries, e.g., those that generate chaos in the classical limit, on PCs. In general, an asymmetric boundary destroys angular momentum conservation and introduces irregular scattering. Theoretical and experimental studies have shown that, similar to the effects of bulk disorder, symmetry breaking of the boundary can result in opening of gaps at the degeneracy points of the energy levels, leading to level repulsion, a typical manifestation of classical chaos. Energy gap opening can diminish AB oscillations through pinning of the corresponding states, leading to vanishing PCs. Since fully chaotic domains are associated with a strong degree of symmetry breaking, PCs are not expected to arise. In nonrelativistic quantum systems governed by the Schrödinger equation, PCs are thus *fragile*.

We addressed the question of whether, in relativistic quantum systems, PCs can arise and sustain in the presence of symmetry-breaking perturbations. Besides the importance of this question to fundamental physics, our work was motivated by the tremendous recent research on two-dimensional Dirac materials such as graphene, topological insulators, molybdenum disulfide (MoS_2), HITP [$\text{Ni}_3(\text{HITP})_2$], and topological Dirac semimetals. The physics of these materials is governed by the Dirac equation. This is thus interest in investigating relativistic PCs in Dirac fermion systems.

Existing theoretical works on relativistic PCs, however, assumed idealized circular-symmetric rings in the ballistic limit. Whether AB oscillations and consequently relativistic PCs can exist in asymmetric rings that exhibit chaos in the classical limit is a fundamental question. Our finding was that, even in the presence of significant boundary deformations so that the classical dynamics becomes fully chaotic, robust PCs can occur in relativistic quantum, Dirac fermion systems, henceforth the term *superpersistent currents* (SPCs). A more striking finding is that SPCs are generated by localized states at the domain boundaries, which are effectively chaotic Dirac whispering gallery modes (WGMs) that carry larger angular momenta. While WGMs are common in photonic systems, its emergence in electronic systems, especially in relativistic quantum systems, is rare and surprising. We developed a physical understanding of the counterintuitive phenomenon of SPCs by analytically exploiting the properties of the spinor wavefunctions in an idealized relativistic quantum system.

The significance of our results lies in the perspective of observing SPCs in the presence of strong random scattering, implying that they may occur in systems of size beyond the mesoscopic limit. This can potentially be a relativistic quantum phenomenon occurring at relatively large scales. There can be significant applications of SPCs in quantum information processing, for which we presented a scheme of Dirac

WGM-based qubit. We note that, previous experimental studies of AB oscillations in graphene and topological insulators made it possible to experimentally test the phenomena of Dirac WGMs and SPCs that we predicted. To motivate experimental verification, we proposed a feasible scheme using three-dimensional topological insulators.

Contrast of eigen-wavefunctions for the Schrödinger particle and Dirac fermion is demonstrated in Fig. 15. For low energy levels, the Schrödinger particle is strongly localized throughout the domain, as shown in Figs. 15(a-c), leading to a flat energy-flux dispersion. However, the Dirac fermion typically travels around the ring's boundaries, forming relativistic WGMs that persist under irregular boundary scattering due to chaos and are magnetic-flux dependent, as shown in Fig. 15(d-f). Conventional wisdom for Schrödinger particle stipulates that asymmetry in the domain geometry can mix/couple well-defined angular momentum states, opening energy gaps and leading to localization of lower states in the entire domain region, so AB oscillations would vanish, as demonstrated both theoretically and experimentally. However, for Dirac fermion, this picture breaks down - there are robust AB oscillations even in the fully chaotic domain and the particle tends to execute motions corresponding to WGMs, leading to SPCs.

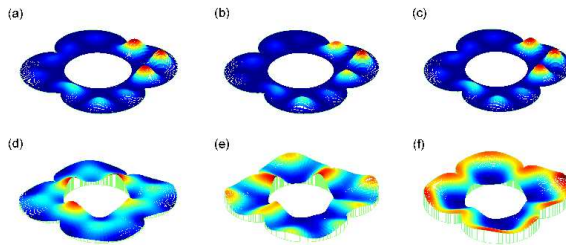


Figure 15: **Superpersistent current in two-dimensional Dirac fermion systems.** Probability distribution of the 10th eigenstate for (a-c) nonrelativistic and (d-f) relativistic AB chaotic billiard for magnetic-flux parameter $\alpha = -1/4, 0, 1/4$, respectively.

Details of this work can be found in

- H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, “Superpersistent currents and whispering gallery modes in relativistic quantum chaotic systems,” *Nature Scientific Reports* **5**, 8963, 1-8 (2015).

4 Personnel Supported and Theses Supervised by PI

4.1 Personnel Supported

The following people received salaries from the AFOSR Project during various time periods.

- **Faculty:** Ying-Cheng Lai (PI), ISS Chair Professor of Electrical Engineering, Professor of Physics.
- **Assistant Research Professor:** Dr. Liang Huang.
- **Ph.D. Students:** Rui Yang (2012), Xuan Ni (2013), Guanglei Wang (ongoing), Lei Ying (ongoing), Hongya Xu (ongoing).

4.2 Ph.D. graduates who participated in research in the project area

1. Rui Yang, Electrical Engineering, ASU, May 2012. Ph.D. Dissertation: *System reconstruction via compressive sensing, complex-network dynamics, and electronic transport in graphene systems.*
2. Xuan Ni, Electrical Engineering, ASU, May 2013. Ph.D. Dissertation: *Effect of chaos on relativistic quantum tunneling*; **Recipient of 2012-2013 Palais Outstanding Doctoral Student Award, ASU ECEE.**

5 Interactions/Transitions

5.1 Collaboration with DoD scientists

- Dr. Vassilios Kovanis from Air Force Research Laboratory at Wright Patterson Air Force Base, on nonlinear and complex dynamical systems.
- Dr. Louis M. Pecora from Naval Research Laboratory in Washington DC, on relativistic quantum transport in Dirac fermion and graphene systems.

5.2 Invited talks on topics derived from the project

During the project period, PI gave the following invited plenary talks, seminars, and colloquia on various topics derived from AFOSR sponsored research.

1. “Transient Chaos,” Colloquium, School of Electrical Engineering and Automation, Tianjin University, Tianjin, China; June 10, 2012.
2. “Chaos-based quantum control,” Undergraduate colloquium, Lanzhou University, Lanzhou, China; July 5, 2012.
3. “Transient chaos,” Plenary talk, Dynamics Days Asia Pacific 7 - The 7th International Conference on Nonlinear Science & the 11th Taiwan International Symposium on Statistical Physics, Academia Sinica, Taipei, Taiwan; August 6, 2012.
4. “Application of chaos in harnessing quantum systems: Modulating quantum transport by transient chaos,” Plenary talk, International Conference on Theory and Applications in Nonlinear Dynamics; Seattle, WA; August 27, 2012.
5. “Research on nonlinear dynamics and complex systems for applied mathematics - a vision,” Distinguished University Lecture (hosted by the President of the University), Kyungpook National University, Daegu, South Korea; September 11, 2012.
6. “Chaos-based quantum control,” Invited talk, minisymposium on Quantum Chaos and Transport, SIAM Conference on Applied Dynamical Systems, Snowbird, UT; May 20, 2013.
7. “Relativistic quantum chaos in Dirac fermion and graphene systems,” Colloquium, Department of Physics and Center for Graphene Research, National University of Singapore; June 26, 2013.
8. “Hidden nodes, extreme events, human-interest dynamics, and quantum entanglement,” Invited talk, Invited-Only ARO Workshop on Information in Complex Dynamical Systems, Burlington, VM; July 18, 2013.
9. “Nonlinear dynamics and complex systems - a mathematical paradigm for cutting-edge, interdisciplinary research,” Keynote lecture, IBS (Institute for Basic Sciences) International Symposium on “Towards a mathematical theory of nonlinear dynamical and complex systems,” Seoul, South Korea; August 1, 2013.
10. “Relativistic quantum chaos in Dirac fermion and graphene systems,” Colloquium, School of Natural and Computing Sciences, University of Aberdeen, Scotland, UK; October 16, 2013.
11. “Graphene: the wonder material for research on relativistic quantum chaos,” Invited talk, The 8th OCPA International Conference on Physics Education and Frontier Physics, Nanyang Technological University, Singapore; June 23, 2014.

12. “Superpersistent currents in chaotic Dirac fermion systems,” Plenary talk, The 13th Experimental Chaos Conference, University of Aberdeen, Scotland, UK; August 27, 2014.
13. “Transient dynamics in nonlinear and complex systems,” Invited talk, ARO Workshop on Cyber Security Dynamics, University of North Carolina, Chapel Hill; September 23, 2014.
14. “Nonlinear dynamics and complex systems - a paradigm for cutting-edge, interdisciplinary research,” Physics Colloquium, Shanxi Normal University, Xi’an, China; March 11, 2015.

6 Past Honors

1. NSF Faculty Career Award, 1997.
2. Air Force PECASE, 1997.
3. Election as a Fellow of the American Physical Society, 1999. Citation: *For his many contributions to the fundamentals of nonlinear dynamics and chaos.*
4. NSF ITR Award, 2003.
5. Outstanding Referee Award, American Physical Society, 2008.

1.

1. Report Type

Final Report

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Ying-Cheng.Lai@asu.edu

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1-480-965-6668

Organization / Institution name

Arizona State University

Grant/Contract Title**The full title of the funded effort.**

RELATIVISTIC QUANTUM TRANSPORT IN GRAPHENE SYSTEMS

Grant/Contract Number**AFOSR assigned control number. It must begin with "FA9550" or "F49620" or "FA2386".**

FA9550-12-1-0095

Principal Investigator Name**The full name of the principal investigator on the grant or contract.**

Ying-Cheng Lai

Program Manager**The AFOSR Program Manager currently assigned to the award**

Dr. Arje Nachman

Reporting Period Start Date

04/15/2012

Reporting Period End Date

04/15/2015

Abstract

The principal Objective of the project was to exploit relativistic quantum manifestations of classical chaos in graphene and two-dimensional Dirac fermion systems. A general discretization method with a proper treatment of the boundary conditions was developed for solving the Dirac equation in closed domains, explicitly demonstrating the phenomenon of relativistic quantum scarring. A conformal mapping method was articulated to solve the eigenvalue problem associated with the Dirac equation for geometric domains with a continuous spectrum of classical behaviors, making it possible to uncover a unique class of relativistic quantum scars: chiral scars. The idea of exploiting chaos to control/modulate quantum transport dynamics was proposed and thoroughly investigated, yielding experimentally feasible schemes of control. The role of classical chaos in regularizing relativistic quantum tunneling dynamics was also studied for both Dirac fermion and graphene systems. The phenomenon of superpersistent currents and whispering gallery modes in chaotic Dirac fermion systems was uncovered and explained. These results elucidated a number of fundamental issues in the emergent field of Relativistic Quantum Chaos, with implications to the development of next generation of nanoscale electronic devices and circuits based on graphene and alternative two-dimensional Dirac materials.

As related works in other interdisciplinary fields, the dynamical mechanism of fractal, branched waves in

optical media (including optical metamaterials) was articulated and analyzed. The interplay between classical nonlinear dynamics and quantum entanglement in optomechanical systems was explored. The phenomenon of low Reynolds number turbulence in ferrofluids was uncovered.

The AFOSR project resulted in 20 refereed-journal papers, including papers in high-impact journals such as Physical Review Letters. The AFOSR support provided PI with the opportunity to supervise a number of PhD students: two graduated, one to graduate in 2016, and two ongoing. PI gave about a dozen plenary lectures, seminars, and colloquiums all over the world on relativistic quantum chaos.

Specific accomplishments include (1) a general method for solving the Dirac equation in closed domains and relativistic quantum scarring, (2) chiral scars in chaotic Dirac fermion systems, (3) relativistic quantum tunneling in Dirac fermion and graphene systems, (4) effect of many-body interactions on relativistic quantum chaos, (5) insensitivity of conductance fluctuations to the nature of classical dynamics, (6) harnessing quantum transport by transient chaos, (7) quantum chaotic scattering in graphene systems without invariant classical dynamics, (8) universal transform for Fano resonance, (9) superpersistent currents and whispering gallery modes in relativistic quantum chaotic systems, (10) emergence of scaling associated with complex branched wave structures in optical media, (11) enhancement of quantum entanglement by nonlinear dynamics in optomechanical systems, and (12) low Reynolds-number turbulence in ferrofluids.

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Archival Publications (published) during reporting period:

1. X. Ni, L. Huang, Y.-C. Lai, and L. M. Pecora, "Effect of chaos on relativistic quantum tunneling," Europhysics Letters 98, 50007, 1-6 (2012). This paper was highlighted in Europhysics News, Vol. 43, No. 6 (2012).
2. L. Ying, L. Huang, Y.-C. Lai, and C. Grebogi, "Conductance fluctuations in graphene systems: The relevance of classical dynamics," Physical Review B 85, 245448, 1-7 (2012).
3. X. Ni, L. Huang, Y.-C. Lai, and C. Grebogi, "Scarring of Dirac fermions in chaotic billiards," Physical Review E 86, 015702, 1-8 (2012).
4. X. Ni, Y.-C. Lai, and W.-X. Wang, "Emergence of scaling associated with complex branched wave structures in optical medium," Chaos 22, 043116, 1-11 (2012).
5. H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, "Chiral scars in chaotic Dirac fermion systems," Physical Review Letters 110, 064102, 1-5 (2013). This work was selected by PRL Editors as an Editors' Suggestion.
6. L. Huang, R. Yang, Y.-C. Lai, and D. K. Ferry, "Lead-position dependent regular oscillations and random fluctuations of conductance in graphene quantum dots," Journal of Physics: Condensed Matters 25, 085502, 1-7 (2013).
7. L. Ying, L. Huang, Y.-C. Lai, and Y. Zhang, "Effect of geometrical rotation on conductance fluctuations in graphene quantum dots," Journal of Physics: Condensed Matters 25, 105802, 1-7 (2013).
8. R. Yang, L. Huang, Y.-C. Lai, C. Grebogi, and L. M. Pecora, "Harnessing quantum transport by transient

chaos," Chaos 23, 013125, 1-9 (2013).

9. G.-L. Wang, L. Ying, Y.-C. Lai, and C. Grebogi, "Quantum chaotic scattering in graphene systems in the absence of invariant classical dynamics," Physical Review E 87, 052908, 1-9 (2013).

10. X. Ni, L. Huang, L. Ying, and Y.-C. Lai, "Relativistic quantum tunneling of Dirac fermions in nonhyperbolic chaotic systems," Physical Review B 87, 224304, 1-10 (2013).

11. G.-L. Wang, L. Huang, Y.-C. Lai, and C. Grebogi, "Nonlinear dynamics and quantum entanglement in optomechanical systems," Physical Review Letters 112, 110406, 1-6 (2014).

12. L. Ying, Y.-C. Lai, and C. Grebogi, "Quantum manifestation of synchronization transition in optomechanical systems," Physical Review A 90, 053810, 1-5 (2014).

13. L. Ying, G.-L. Wang, L. Huang, and Y.-C. Lai, "Quantum chaotic tunneling in graphene systems with electron-electron interactions," Physical Review B 90, 224301, 1-14 (2014).

14. L. Huang, Y.-C. Lai, H.-G. Luo, and C. Grebogi, "Universal formalism of Fano resonance," AIP Advances 5, 017137, 1-18 (2015).

15. Y.-Y. Park, Y.-H. Do, S. Altmeyer, Y.-C. Lai, and G.-W. Lee, "Early effect in time-dependent, high-dimensional nonlinear dynamical systems with multiple resonances," Physical Review E 91, 022906, 1-7 (2015).

16. H.-Y. Xu, L. Huang, Y.-C. Lai, and C. Grebogi, "Superpersistent currents and whispering gallery modes in relativistic quantum chaotic systems," Nature Scientific Reports 5, 8963, 1-8 (2015).

17. S. Altmeyer, Y.-H. Do, and Y.-C. Lai, "Transition to turbulence in Taylor-Couette ferrofluidic flow," Nature Scientific Reports 5, 10781, 1-12 (2015).

18. R. Bao, L. Huang, Y.-C. Lai, and C. Grebogi, "Conductance fluctuations in chaotic bilayer graphene quantum dots," Physical Review E, accepted.

Changes in research objectives (if any):

None

Change in AFOSR Program Manager, if any:

None

Extensions granted or milestones slipped, if any:

None

AFOSR LRIR Number

LRIR Title

Reporting Period

Laboratory Task Manager

Program Officer

Research Objectives

Technical Summary

Funding Summary by Cost Category (by FY, \$K)

	Starting FY	FY+1	FY+2
Salary			
Equipment/Facilities			
Supplies			
Total			

Report Document

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Appendix Documents

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